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ON FIXED POINT THEOREMS AND APPLICATIONS TO PRODUCT OF n-NONLINEAR INTEGRAL OPERATORS IN IDEAL SPACES

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Abstract. The object of the present article is twofold. Firstly, we prove some fixed point theorems for the product of n-given operators need not be Banach algebras, which generalize and extend the existing results. Secondly, we apply the achieved results in proving the existence of solutions for product of n-nonlinear integral equations in ideal spaces (Orlicz spaces and Lebesgue spaces). That results shall be easily applied for numerous problems in different Banach spaces.
Key Words and Phrases: Fixed point theorems, ideal spaces, product of n-integral equations, measure of noncompactness.
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