

## AN INERTIAL SUBGRADIENT-EXTRAGRADIENT ALGORITHM FOR SOLVING PSEUDOMONOTONE VARIATIONAL INEQUALITIES

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**Abstract.** In this paper, we introduce an iteration method for solving pseudomonotone variational inequalities and related pseudoconvex optimization problems in Hilbert spaces. The iterative scheme is based on inertial ideas and subgradient-extragradient ideas. A main feature of the method is that it formally requires only one projection step onto the feasible set. We prove a weak convergence of sequences generated by our method. In the end, some numerical examples are provided to illustrate the effectiveness and performance of the proposed algorithm. Meanwhile, we make some detailed comparisons with the known related schemes.

**Key Words and Phrases:** Variational inequalities, inertial extrapolation, pseudomonotonicity, pseudoconvexity, projection method, subgradient-extragradient method.

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