

THE EXISTENCE OF BEST PROXIMITY POINTS FOR GENERALIZED CYCLIC QUASI-CONTRACTIONS IN METRIC SPACES WITH THE *UC* AND ULTRAMETRIC PROPERTIES

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Abstract. In this paper, in the setting of metric spaces we introduce the notions of generalized cyclic quasi-contractions and the ultrametric property as an applied geometric concept. Then we study the existence and uniqueness of best proximity points for such mappings by using this property and the *UC* property. Also, iterative algorithms are furnished to determine such best proximity points. As a result, we establish a fixed point result and a common fixed point theorem. The presented results extend and improve some recent results in the literature.

Key Words and Phrases: Best proximity point, generalized cyclic quasi-contractions, ultrametric property, *UC* property.

2020 Mathematics Subject Classification: 47H10, 54E05, 54H25.

Acknowledgements. The author would like to thank the anonymous referee for his helpful comments that improved this manuscript.

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Received: January 5, 2021; Accepted: December 17, 2021.