EXISTENCE OF NONTRIVIAL RADIAL SOLUTION FOR SEMILINEAR EQUATION WITH CRITICAL OR SUPERCRITICAL VARIABLE EXPONENT

CHANG-MU CHU AND NING XU

School of Preparatory Education, Guizhou Minzu University,
Guizhou 550025, P.R. China
E-mail: gzmychuchangmu@sina.com. 934019511@qq.com

Abstract. This paper is devoted to study a class of semilinear elliptic equations with critical or supercritical variable exponents. By means of the mountain pass lemma, the existence of a nontrivial radial solution to this problem is obtained.

Key Words and Phrases: Semilinear elliptic equation, variable exponent, nontrivial radial solution, mountain pass lemma.

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