

IMPLICIT AND EXPLICIT VISCOSITY METHODS FOR HIERARCHICAL VARIATIONAL INEQUALITIES ON HADAMARD MANIFOLDS

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Abstract. In this paper, we consider a hierarchical variational inequality problem defined over the set of zeros of a set-valued monotone vector field in the setting of Hadamard manifolds. We also consider bilevel variational inequality problems and bilevel optimization problems as special cases of our variational inequality problem. We develop implicit and explicit viscosity methods for solving our problem for weakly contraction mappings. An inexact version of the explicit viscosity method is also studied. At the end, we provide two examples and computational experiments to illustrate implicit and explicit viscosity methods.

Key Words and Phrases: Hierarchical variational inequality problems; weakly contraction mappings, Hadamard manifolds, monotone vector fields, nonexpansive mappings, bilevel variational inequality problems, bilevel optimization problems.

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