

HYBRID INERTIAL ALGORITHM FOR FIXED POINT AND EQUILIBRIUM PROBLEMS IN REFLEXIVE BANACH SPACES

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Abstract. In this paper, we proposed a hybrid inertial algorithm for approximating fixed points of noncommutative generic 2-generalized Bregman nonspreading mappings with equilibrium in reflexive Banach space. Also, we proved that the sequence generated by such algorithm converges strongly to the common fixed points of such mappings and solved some equilibrium problems in the space. The result established improved and generalized some recently announced results in the literature. A numerical example is given at end of the paper to ascertain some least level of improvement.

Key Words and Phrases: 2-generalized hybrid mapping, normally 2-generalized hybrid mapping, 2-generalized nonspreading mapping, generic 2-generalized Bregman nonspreading mapping, equilibrium problems.

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