ON SYSTEM OF SPLIT GENERALISED MIXED EQUILIBRIUM AND FIXED POINT PROBLEMS FOR MULTIVALUED MAPPINGS WITH NO PRIOR KNOWLEDGE OF OPERATOR NORM

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Abstract. In this paper, we introduce the System of Split Generalized Mixed Equilibrium Problem (SSGMEP), which is more general than the existing well known split equilibrium problem and its generalizations, split variational inequality problem and several other related problems. We propose a new iterative algorithm of inertial form which is independent on the operator norm for solving SSGMEP in real Hilbert spaces. Motivated by the adaptive step size technique and inertial method, we incorporate self adaptive step size and inertial technique to overcome the difficulty of having to compute the operator norm and to accelerate the convergence of the proposed method. Under standard and mild assumptions on the control sequences, we establish the strong convergence of the algorithm, obtain a common solution of the SSGMEP and fixed point of finite family of multivalued demicontractive mappings. We obtain some consequent results which complement several existing results in this direction in the literature. We also apply our results to finding solution of split convex minimisation problems. Numerical example is presented to illustrate the performance of our method as well as comparing it with its non-inertial version.

Key Words and Phrases: Inertial algorithm, system of split generalized mixed equilibrium problems, fixed point problems, multivalued demicontractive mappings, strong convergence.

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