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EXTENSION OF λ-PIR FOR WEAKLY CONTRACTIVE OPERATORS VIA FIXED POINT THEORY

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Abstract. In this article, we apply methods of fixed point theory to investigate a Lambda policy iteration with a randomization algorithm for mappings that are merely weak contractions. As simple examples show, this class of mappings provide a much wider scope than the one afforded by strong contractions usually considered in the literature. More specifically, we investigate the properties of reinforcement learning procedures which have been developed for feedback control, in the framework of fixed point theory. Under fairly general assumptions, we determine sufficient conditions for the convergence with probability one in infinite dimensional policy spaces.

Key Words and Phrases: Fixed point theory, weakly contractive map, Lambda policy iteration with randomization.
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