

FIXED POINT RESULTS IN LOCALLY CONVEX SPACES WITH τ -KREIN-ŠMULIAN PROPERTY AND APPLICATIONS

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Abstract. In this paper, we present some new fixed point theorems in a locally convex space X with the so called τ -Krein-Šmulian property considering the concept of Φ_{Λ}^{τ} -measures of noncompactness, where τ is a weaker Hausdorff locally convex topology of X . Further, we apply our results to discuss the existence of solutions for a nonlinear functional integral equation in the Lebesgue space L^1 .

Key Words and Phrases: Φ_{Λ}^{τ} -measure of noncompactness, τ -sequentially continuous, τ -Krein-Šmulian property, angelic space.

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