

## THE CAUCHY PROBLEM IN SCALE OF BANACH SPACES WITH DEVIATING VARIABLES

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**Abstract.** In this paper, we first prove the existence and uniqueness results for the Cauchy problems in a scale of Banach spaces with deviating variables of the form  $u'(t) = F[t, A(t, u(t)), B(u(h(t)))]$ . We then apply it to study a Cauchy problem for PDEs in a Gevrey class with deviation at the derivatives. This extends some known results.

**Key Words and Phrases:** Scale of Banach spaces, Cauchy problem, deviating variable, Gevrey function, fixed point.

**2020 Mathematics Subject Classification:** 35A10, 34G20, 58D25, 47H10.

**Acknowledgment.** The authors are very grateful to the referees for their careful reading the work that improve the paper. The paper was written when the authors were visiting the Vietnam Institute for Advanced Study in Mathematics (VIASM). The authors thank the institute for its hospitality. This paper is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 101.02-2019.327.

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*Received: January 29, 2019; Accepted: January 28, 2020.*