

## IMPLICIT CONTRACTIONS FOR A SEQUENCE OF MULTI-VALUED MAPPINGS

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**Abstract.** In the present paper, we define a new class of implicit contractions for a sequence of multi-valued mappings on a metric space endowed with a graph to obtain sufficient conditions for existence of common fixed points for this sequence. This will enable us to obtain a simultaneous generalization of various types of fixed point theorems for a sequence of multi-valued mappings. Moreover, multi-fractal operators related to these contractive mappings are considered.

**Key Words and Phrases:** Fixed points, multi-valued mappings, graph.

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