A NEW ALGORITHM FOR SOLVING A SYSTEM OF GENERALIZED MIXED EQUILIBRIUM PROBLEMS AND FINDING COMMON FIXED POINTS OF A FINITE FAMILY OF BREGMAN NONEXPANSIVE MAPPINGS IN BANACH SPACES

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Abstract. In this paper, we introduce a new general iterative algorithm for finding a common element of the set of common fixed points of a finite family of Bregman nonexpansive mappings and the set of solutions of systems of generalized mixed equilibrium problems.
Key Words and Phrases: Banach space, Bregman mapping, fixed point, System of generalized mixed equilibrium, system of mixed variational inequality.
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