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PROPERTIES AND ITERATIVE METHODS FOR THE ELASTIC NET WITH ℓ_p-NORM ERRORS

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Abstract. The p-elastic net (p-EN) with $1 < p < \infty$ is introduced to recover a sparse signal $x \in \mathbb{R}^n$ from $m\,(< n)$ linear measurements with noise. The p-EN, which extends the elastic net of Zou and Hastie [23] and was implicitly suggested by Tropp [16], amounts to minimizing the objective function $(1/p)\|Ax - b\|_p^p + \lambda\|x\|_1 + (\mu/2)\|x\|_2^2$ over $x \in \mathbb{R}^n$, where A is the measurement matrix, b is the observation, and $\lambda > 0$, $\mu > 0$ are regularization parameters. Some basic geometric properties of the p-EN such as how the solution curve of the minimization depends on the parameters λ and μ are investigated. Moreover, iterative algorithms such as the proximal-gradient algorithm and the Frank-Wolfe algorithm are studied for solving the p-EN.

Key Words and Phrases: Lasso, compressed sensing, elastic net, ℓ_p-norm error, proximal gradient, Frank-Wolfe.
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