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ON BEST PROXIMITY PAIRS WITH APPLICATION TO DIFFERENTIAL EQUATIONS

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Abstract. In this paper we consider the following system of differential equations,

$$y' = f(x, y),\ y(x_0) = y_1 \text{ and } z' = g(x, z),\ z(x_0) = z_1,$$

where $f,\ g$ are bounded L^1 functions defined on a rectangle in $\mathbb{R}^2$. We give sufficient conditions for the existence of two functions ϕ and ψ, on an interval I containing x_0, such that

$$|y_1 + \int_{x_0}^{x} f(t, \phi(t))dt - \phi(x)| \leq |y_1 - z_1|,$$

$$|z_1 + \int_{x_0}^{x} g(t, \psi(t))dt - \psi(x)| \leq |y_1 - z_1|$$

for all $x \in I$. To establish the same, we introduce a notation of c-cyclic contractive mapping and prove the existence of best proximity pairs for such a mapping.
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