MEANS AND CONVERGENCE OF SEMIGROUP ORBITS

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Abstract. In this paper we prove the following general theorem. Let $(E, \| \cdot \|_E)$ be a uniformly convex Banach space, and let $C$ be a bounded, closed and convex subset of $E$. Assume that $C$ has nonempty interior and is locally uniformly rotund. Let $F$ be a commutative nonexpansive semigroup acting on $C$. If $F$ has no fixed point in the interior of $C$, then there exists a unique point $\tilde{x}$ on the boundary of $C$ such that each orbit of $F$ converges in norm to $\tilde{x}$. We also establish analogous results for semigroups and mappings which are asymptotically nonexpansive in the intermediate sense.

Key Words and Phrases: Asymptotically nonexpansive in the intermediate sense, fixed point, iterates, locally uniformly rotund set, nonexpansive mapping, semigroup of mappings, uniform convexity.

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References


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