POSITIVE SOLUTION FOR NONLINEAR FRACTIONAL DIFFERENTIAL EQUATION WITH NONLOCAL MULTI-POINT CONDITION

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Abstract. In this paper, we study and consider the positive solution of fractional differential equation with nonlocal multi-point conditions of the from:

$$RLD_0^q u(t) + g(t)f(t, u(t)) = 0, \quad t \in (0, 1)$$
$$u^{(k)}(0) = 0, \quad u(1) = \sum_{i=1}^{m} \beta_i RLD_0^{p_i} u(\eta_i)$$

where $$n - 1 < q < n$$, $$n \geq 2$$, $$n - 1 < p_i < n$$, $$q > p_i$$, $$m \in \mathbb{N}$$, $$k = 0, 1, \ldots, n - 2$$, $$0 < \eta_1 < \eta_2 < \cdots < \eta_m \leq 0$$, $$\beta_i \leq 0$$, $$\kappa \in (0, 1]$$, $$RLD_0^q$$, $$RLD_0^{p_i}$$ are the Riemann-Liouville fractional derivative of order $$q$$, $$p_i$$, $$f : [0, 1] \times C([0, 1], E) \to E$$, $$E$$ be Banach space and $$g : (0, 1) \to \mathbb{R}^+$$ are continuous functions.

The main tools for finding positive solutions of the above problem are the fixed point theorems of Guo-Krasnoselskii and of Boyd and Wong. An example is included to illustrate the applicability of our results.

Key Words and Phrases: Boundary value problems, Riemann-Liouville fractional derivative, fixed point theorems.

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