

## FIXED POINT THEOREMS FOR MULTIVALUED MAPPINGS ON BIPOLAR METRIC SPACES

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**Abstract.** In this article, we introduce concepts of Pompeiu-Hausdorff bipolar metric, multivalued covariant and contravariant contraction mappings in bipolar metric spaces. In addition to these, we express two main fixed point theorems, which are supported with four important corollaries, related to these multivalued mappings. Finally we give an example which presents the applicability of our obtained results.

**Key Words and Phrases:** Bipolar metric space, multivalued contraction, fixed point.

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