

DISCUSSION ON THE EXISTENCE OF BEST PROXIMITY POINTS IN METRIC SPACES

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Abstract. In this paper, we modify the definition of some generalized proximal contractions and enumerate a list of equivalent conditions for various versions of generalized proximal contractions of non-self set-valued mappings on (ordered) metric spaces. By using the fixed point means, we establish the existence of best proximity points for mappings involving such contractions which extend and improve many existing related results, as well as, reveal that most of existing best proximity point theorems on metric spaces are in fact equivalent and immediate consequences of well-known fixed point theorems. Finally, some examples are given to support our results.

Key Words and Phrases: Fixed points, best proximity points, generalized contractions, α -proximal admissible mappings.

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