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ON SOLVING THE VARIATIONAL INEQUALITY AND FIXED POINT PROBLEMS IN q-UNIFORMLY SMOOTH BANACH SPACES

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Abstract. In this research, we focus on two main problems, the first one is a fixed point problem of a nonexpansive semigroup and the other is a variational inequality problem for an inverse strongly accretive mapping. Passing through the modified Mann iterative method, we propose the new iterative scheme to find the common elements solving our mentioned problems. Furthermore, we aim to obtain some strong convergence theorems under certain appropriate conditions in the q-uniformly smooth Banach spaces. Our results improve and extend resulting outcomes in the literature.
Key Words and Phrases: Banach space, fixed point, inverse-strongly accretive mapping, nonexpansive semigroup, q-uniformly smooth, variational inequality.
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