

FIXED POINT THEOREMS IN ORDERED METRIC SPACES AND APPLICATIONS TO NONLINEAR BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, we extend the concept of mixed monotone mappings and then we consider certain fixed point theorems for a pair of mappings in metric spaces with a partial ordering. As an application, we study existence of solutions for the following fourth-order two-point boundary value problems for elastic beam equations:

$$\begin{cases} u''''(t) = f(t, u(t), u''(t)), \\ u(0) = A, u'(0) = B, u''(1) = C, u'''(1) = D, \end{cases}$$

where f is a continuous mapping of $[0, 1] \times \mathbb{R} \times \mathbb{R}$ into \mathbb{R} . Moreover, using these fixed point theorems, we prove several existence results for the solutions of various boundary value problems.

Key Words and Phrases: Fixed point theorem, partially ordered set, boundary value problem, differential equation.

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REFERENCES

- [1] M. Abbas, M. Ali Khan, S. Radenović, *Common coupled fixed point theorems in cone metric spaces for w -compatible mappings*, *Appl. Math. Comput.*, **217**(2010), 195-202.
- [2] A.R. Aftabizadeh, *Existence and uniqueness theorems for fourth-order boundary value problems*, *J. Math. Anal. Appl.*, **116**(1986), 415-426.
- [3] H. Aydi, M. Postolache, W. Shatanawi, *Coupled fixed point results for (ψ, φ) -weakly contractive mappings in ordered G -metric spaces*, *Comput. Math. Appl.*, **63**(2012), 298-309.
- [4] V. Berinde, *Coupled fixed point theorems for Φ -contractive mixed monotone mappings in partially ordered metric spaces*, *Nonlinear Anal.*, **75**(2012), 3218-3228.
- [5] T.G. Bhaskar, V. Lakshmikantham, *Fixed point theorems in partially ordered metric spaces and applications*, *Nonlinear Anal.*, **65**(2006), 1379-1393.
- [6] A. Cabada, S. Lois, *Existence of solution for discontinuous third order boundary value problems*, *J. Comput. Appl. Math.*, **110**(1999), 105-114.
- [7] B.S. Choudhury, A. Kundu, *A coupled coincidence point result in partially ordered metric spaces for compatible mappings*, *Nonlinear Anal.*, **73**(2010), 2524-2531.

- [8] M.A. Del Pino, R.F. Manasevich, *Existence for a fourth-order boundary value problem under a two-parameter nonresonance condition*, Proc. Amer. Math. Soc., **112**(1991), 81-86.
- [9] W.-S. Du, *Coupled fixed point theorems for nonlinear contractions satisfied Mizoguchi-Takahashi's condition in quasiordered metric spaces*, Fixed Point Theory Appl., **2010**(2010), Article ID 876372, 9 pages.
- [10] C.P. Gupta, *Existence and uniqueness theorems for the bending of an elastic beam equation*, Appl. Anal., **26**(1988), 289-304.
- [11] M. Jleli, V. Čojbašić Rajić, B. Samet and C. Vetro, *Fixed point theorems on ordered metric spaces and applications to nonlinear elastic beam equations*, J. Fixed Point Theory Appl., **12**(2012), 175-192.
- [12] Y. Li, *Positive solutions of fourth-order boundary value problems with two parameters*, J. Math. Anal. Appl., **281**(2003), 477-484.
- [13] B. Liu, *Positive solutions of fourth-order two point boundary value problems*, Appl. Math. Comput., **148**(2004), 407-420.
- [14] Z. Liu, J.S. Ume, S.M. Kang, *Positive solutions of a singular nonlinear third order two-point boundary value problem*, J. Math. Anal. Appl., **326**(2007), 589-601.
- [15] N.V. Luong, N.X. Thuan, *Coupled fixed points in partially ordered metric spaces and application*, Nonlinear Anal., **74**(2011), 983-992.
- [16] R. Ma, *Positive solutions of fourth-order two point boundary value problems*, Ann. Differential Equations, **15**(1999), 305-313.
- [17] R. Ma, *Existence of positive solutions of a fourth order boundary value problem*, Appl. Math. Comput., **168**(2005), 1219-1231.
- [18] J.J. Nieto, R.R. López, *Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations*, Order, **22**(2005), 223-239.
- [19] J.J. Nieto, R.R. López, *Existence and uniqueness of fixed point in partially ordered sets and applications to ordinary differential equations*, Acta Math. Sin. (Engl. Ser.), **23**(2007), 2205-2212.
- [20] A. Petruşel, G. Petruşel, *A study of a general system of operator equations in b-metric spaces via the vector approach in fixed point theory*, J. Fixed Point Theory Appl., **9**(2017), 1793-1814.
- [21] I.A. Rus, A. Petruşel, G. Petruşel, *Fixed Point Theory*, Cluj University Press, Cluj-Napoca, 2008.
- [22] B. Samet, C. Vetro, *Coupled fixed point theorems for multi-valued nonlinear contraction mappings in partially ordered metric spaces*, Nonlinear Anal., **74**(2011), 4260-4268.
- [23] B. Samet, C. Vetro, P. Vetro, *Fixed point theorems for α - ψ -contractive type mappings*, Nonlinear Anal., **75**(2012), 2154-2165.
- [24] J.-P. Sun, *Existence of solution and positive solution of BVP for nonlinear third-order dynamic equation*, Nonlinear Anal., **64**(2006), 629-636.
- [25] R.A. Usmani, *A uniqueness theorem for a boundary value problems*, Proc. Amer. Math. Soc., **77**(1979), 329-335.
- [26] D.-B. Wang, J.-P. Sun, *Existence of a solution and a positive solution of a boundary value problem for a nonlinear fourth-order dynamic equation*, Nonlinear Anal., **69**(2008), 1817-1823.
- [27] Y. Yang, *Fourth-order two-point boundary value problems*, Proc. Amer. Math. Soc., **104**(1988), 175-180.
- [28] Q. Yao, *Solution and positive solution for a semilinear third-order two-point boundary value problem*, Appl. Math. Lett., **17**(2004), 1171-1175.

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