THE FIXED POINT PROPERTY FOR CLOSED NEIGHBORHOODS OF LINE SEGMENTS IN $L^p$

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Abstract. We prove that, in $L^p$-spaces with $p \in (1, \infty]$, closed neighborhoods of line segments are dismantlable and hence every monotone operator on these neighborhoods has a fixed point. We also give an example that, for $p = 1$, closed neighborhoods of line segments need not be dismantlable. It is an open question whether every monotone self map of a closed neighborhood of a line segment in $L^1$ has a fixed point.

Key Words and Phrases: Dismantlable ordered set, fixed point property, line segment, closed $L^p$-neighborhood.

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References

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