A NONLOCAL PROBLEM FOR PROJECTED DIFFERENTIAL EQUATIONS AND INCLUSIONS WITH APPLICATIONS

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Abstract. We study a nonlocal problem for projected differential equations and inclusions in finite dimensional spaces. By applying the fixed point theory methods we obtain the existence of solutions to the considered problem for projected differential inclusions. For the case of the projected differential equations we prove, under some suitable conditions, the uniqueness of a solution and the Ulam-Hyers stability of solutions. It is shown how the abstract results can be applied to the study of a market model with the price intervention in the form of price floors and ceilings. An example with exponential demand and supply functions is presented.
Key Words and Phrases: Projected differential inclusion, nonlocal condition, fixed point, Ulam-Hyers stability, market model.
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