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## COMMON SOLUTION TO A SPLIT EQUALITY MONOTONE VARIATIONAL INCLUSION PROBLEM, A SPLIT EQUALITY GENERALIZED GENERAL VARIATIONAL-LIKE INEQUALITY PROBLEM AND A SPLIT EQUALITY FIXED POINT PROBLEM

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**Abstract.** This paper deals with a strong convergence theorem for an iterative method for approximating a common solution to a split equality monotone variational inclusion problem, a split equality generalized general variational-like inequality problem and a split equality fixed point problem for quasi-nonexpansive mappings in real Hilbert spaces. Some consequences are derived from the main result. Finally, we give a numerical example to justify the main result. The main result extends and unifies some recent known results in the literature.

**Key Words and Phrases**:Split equality monotone variational inclusion problem, split equality generalized general variational-like inequality problem, split equality fixed point problem, iterative method.

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