

## APPROXIMATING COINCIDENCE POINTS BY $\alpha$ -DENSE CURVES

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**Abstract.** The purpose of this paper is to show, under suitable conditions, an iterative procedure which if converges, the limit point is a coincidence point of two given itself mappings defined in a subset of a metric space. Also, under additional conditions, the convergence of proposed iterative procedure holds. Our main tool will be the so called  $\alpha$ -dense curves, which allow us to construct such procedure in a stable way, in the specified sense, providing also a bound for the error approximation at each iteration. To justify our result, we will analyze certain integral equations of Volterra type.

**Key Words and Phrases:** Coincidence points, iterative procedures,  $\alpha$ -dense curves.

**2010 Mathematics Subject Classification:** 55M20, 47J25, 47H10.

**Acknowledgements.** The author is grateful to the anonymous referee for his/her useful comments and suggestions. Also, my gratitude to Prof. Dr. G. Mora for his fruitful discussion of the preliminary version of the paper, and to my beloved Loli for her careful reading and grammar corrections to improve the quality of the paper.

### REFERENCES

- [1] M. Abbas, G. Jungck, *Common fixed point results for noncommuting mappings without continuity in cone metric spaces*, J. Math. Anal. Appl., **341**(1)(2008), 416–420.
- [2] A. Alotaibi, V. Kumar, N. Hussain, *Convergence comparison and stability of Jungck-Kirk type algorithms for common fixed point problems*, Fixed Point Theory Appl., **2013**, 2013:173.
- [3] D. Ariza-Ruiz, J. García-Falset, *Iterative approximation to a coincidence point of two mappings*, Appl. Math. Comput., **259**(15)(2015), 762–776.
- [4] A.V. Arutyunov, *An iterative method for finding coincidence points of two mappings*, Comput. Math. Math. Phys., **52**(11)(2012), 1483–1486.
- [5] A. Azam, *Coincidence points of mappings and relations with applications*, Fixed Point Theory Appl., **2012**, 2012:50.
- [6] V. Berinde, *Iterative Approximation of Fixed Points. Second Edition*, Lecture Notes in Mathematics, **1912**, Springer, Berlin, 2007.
- [7] V. Berinde, *Common fixed points of noncommuting discontinuous weakly contractive mappings in cone metric spaces*, Taiwanese J. Math., **14**(5)(2010), 1763–1776.
- [8] Y. Cherruault, G. Mora, *Optimisation globale. Théorie des courbes  $\alpha$ -denses*, Économica, Paris, 2005.

- [9] B.S. Choudhury, S. Kundu, *Extended Ishikawa iteration and its convergence to a coincidence point*, Int. Rev. Pure Appl. Math., **7**(1)(2011), 47–56.
- [10] A. El-Sayed Ahmed, S.A. Ahmed, *Fixed points by certain iterative schemes with applications*, Fixed Point Theory Appl., **2014**, 2014:121.
- [11] G. García, *Interpolation of bounded sequences by alpha-dense curves*, J. Interpolat. Approx. Sci. Comput., **2017**(1)(2017), 1–9.
- [12] J. García-Falset, O. Muñoz Pérez, S. Kishin, *Coincidence problems under contractive type conditions*, Fixed Point Theory, **18**(1)(2017), 213–222.
- [13] K. Goebel, *A coincidence point theorem*, Bull. de L'Acad. Polon. Sci., **16**(9)(1968), 733–735.
- [14] A.M. Harder, T.L. Hicks, *Stability results for fixed point iteration procedures*, Math. Japon., **33**(5)(1988), 693–706.
- [15] A.M. Harder, T.L. Hicks, *A stable iteration procedure for nonexpansive mappings*, Math. Japon., **33**(5)(1988), 687–692.
- [16] N. Hussain *et al.*, *Coincidence point theorems for generalized contractions with application to integral equations*, Fixed Point Theory Appl., **2015**, 2015:78.
- [17] N. Hussain, V. Kumar, R. Chugh, P. Malik, *Jungck-type implicit iterative algorithms with numerical examples*, Filomat, **31**(8)(2017), 2303–2320.
- [18] G. Jungck, *Commuting mappings and fixed points*, Amer. Math. Moth., **83**(4)(1974), 261–263.
- [19] G. Jungck, *Common fixed points for commuting and compatible maps on compacta*, Proc. Amer. Math. Soc., **103**(3)(1988), 977–983.
- [20] A.R. Khan, V. Kumar, N. Hussain, *Analytical and numerical treatment of Jungck-type iterative schemes*, Appl. Math. Comput., **231**(15)(2014), 521–535.
- [21] R. Machuca, *A coincidence problem*, Amer. Math. Moth., **74**(1967), 469.
- [22] R.H. Martin, *Nonlinear Operators and Differential Equations in Banach Spaces*, John Wiley and Sons, 1976.
- [23] G. Mora, *Optimization by space-densifying curves as a natural generalization of the Alienor method*, Kybernetes, **29**(5-6)(2000), 746–754.
- [24] G. Mora, Y. Cherruault, *Characterization and generation of  $\alpha$ -dense curves*, Comput. Math. Appl., **33**(9)(1997), 83–91.
- [25] G. Mora, Y. Cherruault, *The theoretic calculation time associated to  $\alpha$ -dense curves*, Kybernetes, **27**(8)(1998), 919–939.
- [26] G. Mora, Y. Cherruault, *An approximation method for the optimization of continuous functions of  $n$  variables by densifying their domains*, Kybernetes, **28**(2)(1999), 164–180.
- [27] G. Mora, D.A. Redtowitz, *Densifiable metric spaces*, Rev. Real Acad. Cienc. Exactas Fís. Nat. Ser. A Math., **105**(1)(2011), 71–83.
- [28] A.M. Ostrowski, *The round-off stability of iterations*, Z. Angew. Math. Mech., **47**(2)(1967), 77–81.
- [29] J.R. Roshan, V. Parvaneh, I. Altun, *Some coincidence point results in ordered  $b$ -metric spaces and applications in a system of integral equations*, Appl. Math. Comput., **226**(1)(2014), 725–737.
- [30] H. Sagan, *Space-filling Curves*, Springer-Verlag, New York, 1994.
- [31] S.L. Singh, C. Bhatnagar, S.N. Mishra, *Stability of Jungck-type iterative procedures*, Int. J. Math. Math. Sci., **19**(2005), 3035–3043.
- [32] S.L. Singh, B. Prasad, *Some coincidence theorems and stability of iterative procedures*, Comput. Math. Appl., **55**(11)(2008), 2512–2520.

*Received: May 15, 2017; Accepted: September 25, 2017.*

