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FIXED POINT APPROACH TO THE STABILITY OF GENERALIZED POLYNOMIALS

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Dedicated to Professor Ioan A. Rus

Abstract. Using a new fixed point theorem for linear operators which act on function spaces, we give an iterative method for proving the generalized stability in three essential cases and the hyperstability for polynomial equation $\Delta_y^{n+1} f(x) = 0$ on commutative monoids.The proposed iterative fixed point method leads to final concrete unitary estimates, and also improves and complements the known stability results for generalized polynomials.
Key Words and Phrases: Stability, hyperstability, fixed point method, generalized polynomial.
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