TOPOLOGICAL METHOD FOR COUPLED SYSTEMS OF IMPULSIVE STOCHASTIC SEMILINEAR DIFFERENTIAL INCLUSIONS WITH FRACTIONAL BROWNIAN MOTION

T. BLOUHI*, T. CARABALLO** AND A. OUAHAB*

*Laboratory of Mathematics, Univ Sidi Bel Abbes
PoBox 89, 22000 Sidi-Bel-Abbès, Algeria
E-mail: blouhitayeb@yahoo.com, abdelghani.ouahab@univ-sba.dz, agh_ouahab@yahoo.fr

**Depto. Ecuaciones Diferenciales y Análisis Numérico
Universidad de Sevilla, Campus de Reina Mercedes, 41012-Sevilla, Spain
Depart. Mathematics and Computer Science,
Univ of Adrar National Road No. 06, 01000, Adrar, Algeria.
E-mail: caraball@us.es

Abstract. In this paper we prove the existence of mild solutions for a first-order impulsive semilinear stochastic differential inclusion with an infinite-dimensional fractional Brownian motion. We consider the cases in which the right hand side can be either convex or nonconvex-valued. The results are obtained by using two different fixed point theorems for multivalued mappings, more precisely, the technique is based on a multivalued version of Perov's fixed point theorem and a new version of a nonlinear alternative of Leray-Schauder's fixed point theorem in generalized Banach spaces.

Key Words and Phrases: Mild solutions, fractional Brownian motion, impulses, matrix convergent to zero, generalized Banach space, fixed point, set-valued analysis, differential inclusions.

2010 Mathematics Subject Classification: 34A37, 60H15, 60H20, 47H10.

REFERENCES

[1] N.U. Ahmed, *Nonlinear stochastic differential inclusions on Banach space*, Stoch. Anal. Appl., **12**(1994), 1-10.

[2] J.P. Aubin, H. Frankowska, *Set-Valued Analysis*, Birkhauser, Boston, 1990.

[3] D.D. Bainov, P.S. Simeonov, *Systems with Impulsive Effect*, Horwood, Chichester, 1989.

[4] D.D. Bainov, V. Lakshmikantham, P.S. Simeonov, *Theory of Impulsive Differential Equations*, World Scientific, Singapore, 1989.

[5] P. Balasubramaniam, *Existence of solutions of functional stochastic differential inclusions*, Tamkang J. Math., **33**(2002), no. 1, 35-43.

[6] P. Balasubramaniam, S.K. Ntouyas, D. Vinayagam, *Existence of solutions of semilinear stochastic delay evolution inclusions in a Hilbert space*, J. Math. Anal. Appl., **305**(2005), 438-451.

[7] M. Benchohra, J. Henderson, S.K. Ntouyas, *Impulsive Differential Equations and Inclusions*, Hindawi Publishing Corporation, Vol. 2, New York, 2006.

[8] T. Blouhi, T. Caraballo, A. Ouahab, *Existence and stability results for semilinear systems of impulsive stochastic differential equations with fractional Brownian motion*, Stoch. Anal. Appl., **34**(2016), no. 5, 792-834.

[9] T. Blouhi, J.J. Nieto, A. Ouahab, *Existence and uniqueness results for systems of impulsive stochastic differential equations*, Ukrainian Math. J., to appear.

[10] H. Brézis, *Analyse Fonctionnelle*, Théorie et Applications, Masson, Paris, 1983.

[11] T. Caraballo, M. Garrido-Atienza, T. Taniguchi, *The existence and exponential behavior of solutions to stochastic delay evolution equations with a fractional Brownian motion*, Nonlinear Anal., **74**(2011), 3671-3684.

[12] C. Castaing, M. Valadier, *Convex Analysis and Measurable Multifunctions*, Lecture Notes in Mathematics, vol. 580, Springer Verlag, 1977.

[13] O. Cârjă, M. Necula, I.I. Vrabie, *Viability, Invariance and Applications*, North-Holland Mathematics Studies, 207. Elsevier Science B.V., Amsterdam, 2007.

[14] E. Cépa, *Equations Différentielles Stochastiques Multivoques*, Thèse, de doctorat, Université d' Orléans, 1994.

[15] E. Cépa, *Equations Différentielles Stochastiques Multivoques*, Séminaire Probabilités XXIX 86107, Springer, Berlin, 1995.

[16] G. Da Prato, J. Zabczyk, *Stochastic Equations in Infinite Dimensions*, Cambridge University Press, Cambridge, 1992.

[17] K. Deimling, *Multi-valued Differential Equations*, W. De Gruyter, Berlin, New York, 1992.

[18] S. Djebali, L. Górniewicz, A. Ouahab, *First order periodic impulsive semilinear differential inclusions: existence and structure of solution sets*, Math. and Comput. Mod., **52**(2010), 683-714.

[19] J. Dugundji, A. Granas, *Fixed Point Theory*, Springer-Verlag, New York, 2003.

[20] T.C. Gard, *Introduction to Stochastic Differential Equations*, Marcel Dekker, New York, 1988.

[21] I.I. Gikhman, A. Skorokhod, *Stochastic Differential Equations*, Springer-Velag, 1972.

[22] J.R. Graef, J. Henderson, A. Ouahab, *Impulsive differential inclusions. A Fixed Point Approach*, De Gruyter Series in Nonlinear Analysis and Applications 20. Berlin, de Gruyter, 2013.

[23] J.R. Graef, A. Ouahab, *Structure of solutions sets and a continuous version of Filippov's Theorem for first order impulsive differential inclusions with periodic conditions*, Electronic J. Qual., **24**(2009), 1-23.

[24] C. Guilan, H. Kai, *On a type of stochastic differential equations driven by countably many Brownian motions*, J. Funct. Anal., **203**(2003), 262-285.

[25] L. Górniewicz, *Topological Fixed Point Theory of Multi-Valued Mappings*, Mathematics and its Applications, vol. 495, Kluwer Academic Publ., Dordrecht, 1999.

[26] B.S. Jensen, *The Dynamic Systems of Basic Economic Growth Models*, Kluwer, Dordrecht, 1994.

[27] M. Kisielewicz, *Stochastic Differential Inclusions and Applications*, Springer Optimization and Its Applications, 80, Springer, New York, 2013.

[28] A. Lasota, Z. Opial, *An application of the Kakutani-Ky Fan theorem in the theory of ordinary differential equations*, Bull. Acad. Pol. Sci. Ser. Sci. Math. Astron. Phys., **13**(1965), 781-786.

[29] X. Mao, *Stochastic Differential Equations and Applications*, Horwood, Chichester, 1997.

[30] J.D. Murray, *Mathematical Biology*, Springer, Berlin, 1989.

[31] J. Musielak, *Introduction to Functional Analysis*, PWN, Warszawa, 1976 (in Polish).

[32] H.W. Ning, B. Liu, *Existence results for impulsive neutral stochastic evolution inclusions in Hilbert Space*, Acta Mathematica Sinica, **27**(2011), no. 7, 1405-1418.

[33] D. Nualart, *The Malliavin Calculus and Related Topics*, Second Edition, Springer-Verlag, Berlin, 2006.

[34] B. Øksendal, *Stochastic Differential Equations: An Introduction with Applications*, Fourth Edition, Springer-Verlag, Berlin, 1995.

[35] A. Ouahab, *Some Perov's and Karsnosel'skii type fixed point results and application*, Commun. Applied Analysis, **19**(2015), 623-642.

[36] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer-Verlag, New York, 1983.

[37] I.R. Petre, A. Petruşel, *Krasnoselskii's theorem in generalized Banach spaces and applications*, Electron. J. Qual. Theory Differ. Equ., **85**(2012), 20 pp.

[38] R. Precup, *The role of matrices that are convergent to zero in the study of semilinear operator systems*, Math. Comp. Modelling, **49**(2009), 703-708.

[39] I.A. Rus, *The theory of a metrical fixed point theorem: theoretical and applicative relevances*, Fixed Point Theory, **9** (2008), 541-559.

[40] M.L. Sinacer, J.J. Nieto, A. Ouahab, *Random fixed point theorem in generalized Banach space and applications*, Random Oper. Stoch. Eq., **24**(2016), 93-112.

[41] H. Sobczyk, *Stochastic Differential Equations with Applications to Physics and Engineering*, Kluwer Academic Publishers, London, 1991.

[42] C.P. Tsokos, W.J. Padgett, *Random Integral Equations with Applications to Life Sciences and Engineering*, Academic Press, New York, 1974.

[43] I.I. Vrabie, C_0-*semigroups and applications*, North-Holland Mathematics Studies, 191. North-Holland Publishing Co., Amsterdam, 2003.

[44] D. Wagner, *Survey of measurable selection theorems*, SIAM J. Control Optim., **15**(1977), 859-903.

[45] S.J. Wu, X.L. Guo, S.Q. Lin, *Existence and uniqueness of solutions to random impulsive differential systems*, Acta Math. Appl. Sinica, **22**(2006), 595-600.