

## ITERATIVE METHODS FOR SYSTEM OF VARIATIONAL INCLUSIONS INVOLVING ACCRETIVE OPERATORS AND APPLICATIONS

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**Abstract.** The purpose of this paper is to study existence and approximation of solutions of system of variational inclusions involving multi-valued  $H$ -accretive and single-valued accretive operators over two different closed convex subsets of a Banach space. The convergence analysis of two proposed iteration processes for approximating solutions will be conducted within the conceptual framework of the “altering point technique” without uniform convexity of underlying spaces. This technique should make existing or new results in solving system of variational inequalities and variational inclusions.

**Key Words and Phrases:** Accretive operator, altering points, Mann iteration method, Lipschitz mapping, resolvent operator, variational inclusion problem.

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