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POSITIVE SOLUTIONS FOR A SYSTEM OF p-LAPLACIAN BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, we investigate the existence of positive solutions for a system of fourth order p-Laplacian boundary value problems

$$\begin{cases} -((-x''')^{p-1})' = f(t, x, x', y, y'), t \in [0, 1], \\ -((-y''')^{p-1})' = g(t, x, x', y, y'), t \in [0, 1], \\ x(0) = x'(1) = x''(0) = x'''(1) = 0, \\ y(0) = y'(1) = y''(0) = y'''(1) = 0, \end{cases}$$

where $p > 1$, $f, g \in C([0, 1] \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+, \mathbb{R}^+)(\mathbb{R}^+ := [0, \infty))$. Under some new general conditions on f and g, we use the fixed point index to establish two existence theorems for the above system. The interesting point lies in the fact that the nonlinear term f, g can be allowed to depend on the first derivative of the unknown functions, and this derivative dependence in systems is seldom considered in the literature.
Key Words and Phrases: p-Laplacian equation; positive solution; fixed point index; derivative dependence.
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References

[1] L. Cheng, W. Liu, Q. Ye, *Boundary value problem for a coupled system of fractional differential equations with p-Laplacian operator at resonance*, Electron. J. Diff. Equ., **2014**(2014), no. 60, 1–12.

[2] W. Jiang, *Solvability for a coupled system of fractional differential equations at resonance*, Nonlinear Anal. Real World Appl., **13**(2012), no. 5, 2285–2292.

[3] J. Jiang, L. Liu, Y. Wu, *Positive solutions for p-Laplacian fourth-order differential system with integral boundary conditions*, Discrete Dyn. Nat. Soc., **2012**(2012), Article ID 293734, 19 pages.

[4] Y. Li, Z. Wei, *Positive solutions for a coupled system of mixed higher-order nonlinear singular fractional differential equations*, Fixed Point Theory, **15**(2014), no. 1, 167–178.

[5] W. Yang, *Positive solutions for nonlinear semipositone fractional q-difference system with coupled integral boundary conditions*, Appl. Math. Comput., **244**(2014), 702–725.

[6] N. Xu, W. Liu, *Iterative solutions for a coupled system of fractional differential-integral equations with two-point boundary conditions*, Appl. Math. Comput., **244**(2014), 903–911.

[7] Z. Yang, *Positive solutions for a system of p-Laplacian boundary value problems*, Comput. Math. Appl., **62**(2011), no. 12, 4429–4438.

[8] J. Xu, Z. Yang, *Positive solutions for a system of nth-order nonlinear boundary value problems*, Electron. J. Qual. Theory Differ. Equ., **2011**(2011), no. 4, 1–16.

[9] J. Xu, Z. Yang, *Positive solutions for a system of generalized Lidstone problems*, J. Appl. Math. Comput., **37**(2011), no. 1-2, 13–35.

[10] J. Xu, Z. Yang, Three positive solutions for a system of singular generalized lidstone problems, Electron. J. Diff. Equ., **2009**(2009), no. 163, 1–9.

[11] R. Precup, *A vector version of Krasnosel'skii's fixed point theorem in cones and positive periodic solutions of nonlinear systems*, J. Fixed Point Theory Appl., **2**(2007), no. 1, 141–151.

[12] G. Infante, M. Maciejewski, R. Precup, *A topological approach to the existence and multiplicity of positive solutions of (p, q)-Laplacian systems*, arXiv:1401.1355.

[13] Y. Ding, J. Xu, X. Zhang, *Positive solutions for a $2n$th-order p-Laplacian boundary value problem involving all derivatives*, Electron. J. Diff. Equ., **2013**(2013), no. 36, 1–14.

[14] D. Guo, V. Lakshmikantham, *Nonlinear Problems in Abstract Cones*, Academic Press, Orlando, 1988.

[15] I.A. Rus, A. Petruşel, M.A. Şerban, *Weakly Picard operators: equivalent definitions, applications and open problems*, Fixed Point Theory, **7**(2006), no. 1, 3-22.

[16] I.A. Rus, A. Petruşel, G. Petruşel, *Fixed Point Theory*, Cluj University Press, 2008.

[17] D. O'Regan, N. Shahzad, R.P. Agarwal, *Fixed point theory for generalized contractive maps on spaces with vector-valued metrics*, Fixed Point Theory and Applications, (Eds. Y.J. Cho, J.K. Kim, S.M. Kang), Vol. 6, Nova Sci. Publ., New York, 2007, 143-149.

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