# EXISTENCE THEOREMS OF A NEW SET-VALUED MT-CONTRACTION IN $b$-METRIC SPACES ENDOWED WITH GRAPHS AND APPLICATIONS

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**Abstract.** A new concept of set-valued Mizoguchi-Takahashi $G$-contractions is introduced in this paper and some fixed point theorems for such mappings in $b$-metric spaces endowed with directed graphs are established under some sufficient conditions. Our results improve and extend those of [20] and [24]. We also give some examples supporting our main results. As an applications, we prove the existence of fixed points for multivalued mappings satisfying generalized MT-contractive condition in $\epsilon$-chainable $b$-metric spaces and the existence of a solution for some integral equations.
**Key Words and Phrases**: Fixed point, Mizoguchi-Takahashi function, $b$-metric spaces, directed graph, set-valued map, integral equation, $\epsilon$-chainable metric space.
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