

FIXED POINT THEOREMS IN QUASI-METRIC SPACES AND THE SPECIALIZATION PARTIAL ORDER

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Abstract. In this paper we present a new fixed point theorem in quasi-metric spaces which captures the spirit of Kleene's fixed point theorem. To this end, we explore the fundamental assumptions in the aforesaid result when we consider quasi-metric spaces endowed with the specialization partial order. Thus, we introduce an appropriate notion of order-completeness and order-continuity that ensure the existence of fixed point with distinguished properties. Moreover, some fixed point theorems are derived as a particular case of our main result when the self-mappings under consideration satisfy, in addition, any type of Banach contractive condition under different quasi-metric notions of completeness.

Key Words and Phrases: quasi-metric, specialization partial order, order-completeness, fixed point, monotonicity, order-continuity, contraction.

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