FIXED POINT THEOREMS IN GENERALIZED BANACH ALGEBRAS AND APPLICATIONS

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Abstract. In this paper, we prove some fixed point theorems in vector algebra Banach spaces. We establish the versions of Perov, Schauder and Krasnosel'skki type fixed point theorem for the sum of a contraction operator and a compact operator. The obtained results are applied to prove some theorems on the existence of solutions to nonlinear integral equations in Banach algebras. Finally, some example are given to illustrate the result.

Key Words and Phrases: Generalized Banach space, algebra Banach space, fixed point, multi-valued map, matrix, fractional integral equation.

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