Fixed Point Theory, 19(2018), No. 2, 681-706
DOI: 10.24193/fpt-ro.2018.2.53
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

COMMON FIXED POINTS OF ASYMPTOTICALLY REGULAR SEMIGROUPS EQUIPPED WITH GENERALIZED LIPSCHITZIAN CONDITIONS

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Abstract. We use the generalized Lipschitzian condition as employed by Imdad and Soliman [M. Imdad, A.H. Soliman, *On uniformly generalized Lipschitzian mappings*, Fixed Point Theory Appl., **2010**(2010), Article ID 692401] to extend the Lipschitzian condition which was used on the common fixed point theorems for one parameter semigroups of asymptotically regular mappings in Banach spaces by several authors before. Our results extend some relevant common fixed point theorems due to the works of Górnicki [J. Górnicki, *Fixed points of asymptotically regular semigroups in Banach spaces*, Rend. Circ. Mat. Palermo (2), **XLVI**(1997), 89-118] and Wiśnicki [A. Wiśnicki, *On the structure of fixed-point sets of asymptotically regular semigroups*, J. Math. Anal. Appl., **393**(2012), 177-184].
Key Words and Phrases: Asymptotically regular semigroup, generalized Lipschitzian semigroup, common fixed point, weakly convergent sequence coefficient, Opial condition, uniformly convex Banach space.
2010 Mathematics Subject Classification: 47H10, 47H20.

Acknowledgement. We would like to thank the referees for his/her comments on the manuscript.

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Received: February 21, 2016; Accepted: June 2, 2016.