STRONG CONVERGENCE OF PROJECTED REFLECTED GRADIENT METHODS FOR VARIATIONAL INEQUALITIES

PAUL-EMILE MAINGÉ

University of Antilles, Martinique, French West Indies

E-mail: Paul-Emile.Mainge@martinique.univ-ag.fr

Abstract. The purpose of this work is to revisit the numerical approach to classical variational inequality problems, with monotone and Lipschitz continuous mapping, by means of a regularized dynamical method. A main feature of the method is that it formally requires only one projection step onto the feasible set and only one evaluation of the involved mapping (at each iteration), combined with some viscosity-like regularization process. A strong convergence theorem is established in a general setting that allows the use of varying step-sizes without any requirement of additional projections. We also point out that the considered method in absence of regularization does not generate a Fejer-monotone monotone sequence. So a new analysis is developed for this purpose.

Key Words and Phrases: Variational inequality, monotone operator, dynamical-type method, strong convergence, regularization process, viscosity method.

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