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PERIODIC SOLUTIONS OF A CLASS OF NONLINEAR PLANAR SYSTEMS

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Abstract. In this paper, by the use of a generalized version of the Poincaré-Birkhoff fixed point theorem due to Franks, we prove the existence of at least two geometrically distinct periodic solutions for a class of nonlinear planar systems, and at least one of them is unstable.
Key Words and Phrases: Nonlinear planar system, exact symplectic map, boundary twist condition, Poincaré-Birkhoff fixed point theorem.
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