

## ON THE CLASS OF RELATIVELY WEAKLY DEMICOMPACT NONLINEAR OPERATORS

BILEL KRICHEN\* AND DONAL O'REGAN\*\*

\*Department of Mathematics,  
Faculty of Sciences of Sfax, University of Sfax,  
Road Soukra Km 3.5 B.P. 1171, 3000, Sfax Tunisia  
E-mail: krichen\_bilel@yahoo.fr

\*\*School of Mathematics, National University of Ireland  
Galway, Ireland  
E-mail: donal.oregan@nuigalway.ie

**Abstract.** In this paper we discuss some topological properties of the set

$$\mathcal{F}(S_0, T, z) := \{x \in X : S_0x \in Tx + z\},$$

where  $T$  is a nonlinear multi-valued mappings and  $S_0$  is a single-valued mappings acting on a Banach space  $X$ . This study is based on a new concept, the so called weakly relative demicompactness for nonlinear operators.

**Key Words and Phrases:** Demicompact operator, multi-valued mapping, weak topology, measure of weak noncompactness.

**2010 Mathematics Subject Classification:** 58C07, 47H04.

### REFERENCES

- [1] W.Y. Akashi, *On the perturbation theory for Fredholm operators*, Osaka J. Math., **21**(1984), 603-612.
- [2] J. Appell, E. De Pascale, *Su alcuni parametri connessi con la misura di non compattezza di Hausdorff in spazi di funzioni misurabili*, Boll. Unione Mat. Ital. Sez. B, **6**(1984), 497-515.
- [3] Z. Artstein, *Continuous dependence of solutions of operator equations*, Trans. Amer. Math. Soc., **231**(1977), no. 1, 143-166.
- [4] J. Banas, J. Rivero, *On measures of weak noncompactness*, Ann. Mat. Pura Appl., **151**(1988), 213-224.
- [5] W. Chaker, A. Jeribi, B. Krichen, *Demicompact linear operators, essential spectrum and some perturbation results*, Math. Nachr., **288**(2015), 1476-1486.
- [6] F.S. De Blasi, *On a property of the unit sphere in a Banach space*, Bull. Math. Soc. Sci. Math. R.S. Roumanie, (N.S.), **21**(1977), 259-262.
- [7] L. Gorniewicz, *Topological Fixed Point Theory of Multivalued Mappings*, Springer, Berlin, 2009.
- [8] A. Jeribi, B. Krichen, *Nonlinear Functional Analysis in Banach Spaces and Banach Algebras: Fixed Point Theory under Weak Topology for Nonlinear Operators and Block Operator Matrices with Applications*, Monographs and Research Notes in Mathematics, CRC Press Taylor and Francis, 2015.

- [9] B. Krichen, *Relative essential spectra involving relative demicompact unbounded linear operators*, Acta Math. Sci. Ser. B Engl., **34**(2014), 546-556.
- [10] E. Megginson, Robert, *An Introduction to Banach Space Theory*, Graduate Texts in Mathematics, Springer Verlag, 1988.
- [11] Z. Opial, *Nonexpansive and monotone mappings in Banach spaces*, Center for Dynamical Systems, Brown Univ, Providence, R.I, 1967, 1-67.
- [12] W.V. Petryshyn, *Construction of fixed points of demicompact mappings in Hilbert space*, J. Math. Anal. Appl., **14**(1966), 276-284.
- [13] W.V. Petryshyn, *Remarks on condensing and  $k$ -set-contractive mappings*, J. Math. Anal. Appl., **39**(1972), 717-741.
- [14] W.V. Petryshyn, *Structure of the fixed points sets of  $k$ -set-contractions*, Arch. Rational Mech. Anal., **40**(1971), 312-328.

*Received: January 14, 2016; Accepted: March 12, 2016.*