

A COUPLED FIXED POINT PROBLEM UNDER A FINITE NUMBER OF EQUALITY CONSTRAINTS IN A BANACH SPACE PARTIALLY ORDERED BY A CONE

MOHAMED JLELI* AND BESSEM SAMET**

*Department of Mathematics, College of Science
King Saud University, P.O. Box 2455, Riyadh, 11451, Saudi Arabia
E-mail: jleli@ksu.edu.sa

**Department of Mathematics, College of Science
King Saud University, P.O. Box 2455, Riyadh, 11451, Saudi Arabia
E-mail: bsamet@ksu.edu.sa

Abstract. Let $(E, \|\cdot\|)$ be a Banach space with a cone P . Let $F, \varphi_i : E \times E \rightarrow E$ ($i = 1, 2, \dots, r$) be a finite number of mappings. We obtain sufficient conditions for the existence and uniqueness of solutions to the problem

$$\begin{cases} F(x, y) &= x, \\ F(y, x) &= y, \\ \varphi_i(x, y) &= 0_E, \quad i = 1, 2, \dots, r, \end{cases}$$

where 0_E is the zero vector of E .

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