# ON THE SIZE OF A MAP

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**Abstract.** Some properties depending on an upper bound of the diameter of fibers of a continuous map $f$ from the $n$-dimensional unit cube $I^n$ to the Euclidean space are investigated. In particular, we consider the problem when the image $f(I^n)$ has the nonempty interior. Obtained results are consequences of the Poincaré theorem and some theorems on extensions of maps. Generalizations of the De Marco theorem and the Borsuk theorem are presented.

**Key Words and Phrases**: Domain invariance theorem, Bolzano-Poincaré theorem, Brouwer fixed point theorem, size of a map.

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