

FIXED POINTS OF HAMMERSTEIN-TYPE EQUATIONS ON GENERAL CONES

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Abstract. We obtain new results on the existence and multiplicity of fixed points of Hammerstein equations in very general cones. In order to achieve this, we combine a new formulation of cones in terms of continuous functionals with fixed point index theory. Many examples and an application to boundary value problems are also included.

Key Words and Phrases: Cones, fixed points, Hammerstein equations.

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