

ON GENERALIZED COUPLED FIXED POINTS WITH APPLICATIONS TO THE SOLVABILITY OF COUPLED SYSTEMS OF NONLINEAR QUADRATIC INTEGRAL EQUATIONS

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Abstract. In this paper, we introduce the notion of generalized coupled fixed points and, as a consequence of Darbo's fixed point theorem associated to an abstract measure of noncompactness, we present a result about the existence of this class of coupled fixed points in Banach spaces. As an application, we investigate the existence of solutions for a class of coupled systems of nonlinear quadratic integral equations of Volterra type.

Key Words and Phrases: Measures of noncompactness, generalized coupled fixed point, coupled system, integral equation, fixed point theorem.

2010 Mathematics Subject Classification: 45G10, 45M99, 47H09.

Acknowledgment. The second author was partially supported by the project MTM2013-44357-P.

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Received: May 20, 2015; Accepted: March 1st, 2016.