

HYBRID VISCOSITY EXTRAGRADIENT METHOD FOR SYSTEMS OF VARIATIONAL INEQUALITIES, FIXED POINTS OF NONEXPANSIVE MAPPINGS, ZERO POINTS OF ACCRETIVE OPERATORS IN BANACH SPACES

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Abstract. In this paper, we introduce a hybrid viscosity extragradient method for a general system of variational inequalities with solutions being also common fixed points of a countable family of nonexpansive mappings and zeros point of an accretive operator in real smooth Banach spaces. Under quite appropriate assumptions, we obtain some strong convergence results which improve and develop the corresponding results in the literature.

Key Words and Phrases: Hybrid viscosity extragradient method, general system of variational inequalities, accretive operator, nonexpansive mapping.

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