

## $\alpha$ -TYPE FUZZY $\mathcal{H}$ -CONTRACTIVE MAPPINGS IN FUZZY METRIC SPACES

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**Abstract.** We introduce a new concept of  $\alpha$ -fuzzy  $\mathcal{H}$ -contractive mapping which is essentially weaker than the class of fuzzy contractive mapping and stronger than the concept of  $\alpha$ - $\phi$ -fuzzy contractive mapping. For this type of contractions, the existence and uniqueness of fixed point in fuzzy  $M$ -complete metric spaces is also established.

**Key Words and Phrases:** Contractive mapping, fixed point, t-norm, fuzzy metric space, strong fuzzy metric space.

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