

## FIXED POINT THEOREMS FOR THE SUM OF TWO OPERATORS ON UNBOUNDED CONVEX SETS AND AN APPLICATION

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**Abstract.** In this paper, we establish new fixed point results for the sum of two operators  $A$  and  $B$ , where the operator  $A$  is assumed to be weakly compact and (ws)-compact, while  $B$  is a weakly condensing and expansive operator defined on unbounded domains under different boundary conditions as well as other additional assumptions. In addition, we get new generalized forms of the Krasnosel'skii fixed point theorem in a Banach space by using the concept of measure of weak noncompactness of De Blasi. Later on, we give an application to solve a nonlinear Hammerstein integral equation in  $L^1$ -space.

**Key Words and Phrases:** (ws)-compact, weakly condensing, expansive operator, measure of weak noncompactness, fixed point theorems.

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