

FIXED POINTS AND COMPACT WEIGHTED COMPOSITION OPERATORS ON BANACH WEIGHTED HARDY SPACES

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Abstract. We consider weighted composition operators $C_{\psi, \varphi}^r$ acting on Banach weighted Hardy spaces in the open unit disk such that the norms of the kernel functions for the appropriate order derivatives tend to infinity as one approaches the boundary. We investigate the relation between the compactness of $C_{\psi, \varphi}^r$, the angular derivatives and the fixed points of φ , and we will see that compactness of $C_{\psi, \varphi}^r$ for some weight functions ψ forces φ to have a fixed point inside the open unit disk.

Key Words and Phrases: Banach weighted Hardy spaces, bounded point evaluation, weighted composition operator, Denjoy-Wolff point, fixed point.

2010 Mathematics Subject Classification: 47B33, 47B37.

Acknowledgments. The authors are grateful to the referees for careful reading of the manuscript. Their remarks motivated the authors to make some valuable improvements.

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Received: October 12, 2015; Accepted: January 19, 2017.

