

**THE SHRINKING PROJECTION METHOD  
FOR A FINITE FAMILY OF DEMIMETRIC MAPPINGS  
WITH VARIATIONAL INEQUALITY PROBLEMS  
IN A HILBERT SPACE**

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**Abstract.** In this paper, using a new nonlinear mapping called demimetric and the shrinking projection method, we prove a strong convergence theorem for finding a common element of the set of common fixed points for a finite family of these new demimetric mappings and the set of common solutions of variational inequality problems for a finite family of inverse strongly monotone mappings in a Hilbert space. Using the result, we obtain well-known and new strong convergence theorems in a Hilbert space.

**Key Words and Phrases:** Fixed point, demimetric mapping, inverse strongly monotone mapping, shrinking projection method, variational inequality problem.

**2010 Mathematics Subject Classification:** 47H05, 47H10.

**Acknowledgements.** The first author was partially supported by Grant-in-Aid for Scientific Research No. 15K04906 from Japan Society for the Promotion of Science. The second author was partially supported by the grant MOST 105-2115-M-037-001. The third author was partially supported by the grant MOST 105-2115-M-039-002-MY3.

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*Received: July 28, 2016; Accepted: June 14, 2017.*

