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CONVERGENCE ANALYSIS OF COMMON SOLUTION OF CERTAIN NONLINEAR PROBLEMS

F.U. OGBUISI* AND O.T. MEWOMO**

*School of Mathematics, Statistics and Computer Science
University of KwaZulu-Natal
Durban, South Africa
E-mail: 215082189@stu.ukzn.ac.za fudochukwu@yahoo.com

**School of Mathematics, Statistics and Computer Science
University of KwaZulu-Natal
Durban, South Africa
E-mail:mewomoo@ukzn.ac.za

Abstract. We introduce an iterative algorithm for approximating a common fixed point of an infinite family of left Bregman strongly nonexpansive mappings which is also a common solution of a finite system of generalized mixed equilibrium problems and a common zero of a finite family of maximal monotone operators in a reflexive real Banach space. A strong convergence theorem is also proved for finding an element in the intersection of the set of solution of a fixed point problem for infinite family of left Bregman strongly nonexpansive mappings, the set of solutions of a system of generalized mixed equilibrium problems and the set of zero points of a finite family of maximal monotone operators in a reflexive real Banach space. The result of this paper complement many related and important results in the literature.
Key Words and Phrases: Bregman distance, Bregman projection, maximal monotone operator, generalized mixed equilibrium problem, resolvent, Legendre function, reflexive real Banach space, zero point.
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References

[1] R.P. Agarwal, J.W. Chen, Y.J. Cho, Z. Wan, *Stability analysis for parametric generalized vector quasivariational-like inequality problems*, J. Inequal. Appl., **57**(2012) 15 pp.
[2] Y.I. Alber, *Generalized projection operators in Banach spaces: properties and applications*, In: Proceedings of the Israel Seminar Ariel, Israel, Function Differential Equation, **1**(1994), 1-21.

[3] Y.I. Alber, *Metric and generalized projection operators in Banach spaces: properties and applications*, In Kartsatos, A.G. (ed.) Theory and Applications of Nonlinear Operators of Monotonic and Accretive Type, M. Dekker New York, 1996, 15-50.
[4] Y.I. Alber, D. Butnariu, *Convergence of Bregman projection methods for solving consistent convex feasibility problems in reflexive Banach spaces*, J. Optim. Theory Appl., **92**(1997), 33-61.
[5] H.H. Bauschke, J.M. Borwein, *Legendre functions and the method of random Bregman projections*, J. Convex Anal., **4**(1997), 27-67.
[6] H.H. Bauschke, J.M. Borwein, P.L. Combettes, *Essential smoothness, essential strict convexity, and Legendre functions in Banach spaces*, Commun. Contemp. Math., **3**(2001), 615-647.
[7] H.H. Bauschke, J.M. Borwein, P.L. Combettes, *Bregman monotone optimization algorithms*, SIAM J. Control Optim., **42**(2003), 596-636.
[8] H.H. Bauschke, A.S. Lewis, *Dykstra's algorithm with Bregman projections: a convergence proof*, Optimization, **48**(2000), 409-427.
[9] E. Blum, W. Oettli, *From optimization and variational inequalities to equilibrium problems*, Math. Student, **63**(1994), 123-145.
[10] J.F. Bonnans, A. Shapiro, *Perturbation Analysis of Optimization Problems*, Springer, New York, 2000.
[11] J.M. Borwein, S. Reich, S. Sabach, *A characterization of Bregman firmly nonexpansive operators using a new monotonicity concept*, J. Nonlinear Convex Anal., **12**(2011), 161-184.
[12] L.M. Bregman, *The relaxation method for finding common points of convex sets and its application to the solution of problems in convex programming*, USSR Comput. Math. Math. 17 Phys., **7**(1967), 200-217.
[13] H. Brézis, P.L. Lions, *Produits infinis de résolventes*, Israel J. Math., **29**(1978), 329-345.
[14] R.S. Burachik, A.N. Iusem, *A generalized proximal point algorithm for the variational inequality problem in Hilbert space*, SIAM J. Optim., **8**(1998), 197-216.
[15] D. Butnariu, Y. Censor, S. Reich, *Iterative averaging of entropic projections for solving stochastic convex feasibility problems*, Comput. Optim. Appl., **8**(1997), 21-39.
[16] D. Butnariu, A.N. Iusem, *Totally Convex Functions for Fixed Points Computation and Infinite Dimensional Optimization*, Applied Optimization, vol. 40, Kluwer Academic, Dordrecht, 2000.
[17] D. Butnariu, A.N. Iusem, C. Zalinescu, *On uniform convexity, total convexity and con-vergence of the proximal point and outer Bregman projection algorithms in Banach spaces*, J. Convex Anal., **10**(2003), 35-61.
[18] D. Butnariu, G. Kassay, *A proximal-Projection method for finding zeroes of set valued operators*, SIAM J. Control Optim., **47**(2008), 2096-2136.
[19] D. Butnariu, E. Resmerita, *Bregman distances, totally convex functions, and a method for solving operator equations in Banach spaces*, Abstr. Appl. Anal., (2006), 1-39.
[20] R.S. Burachik, *Generalized proximal point methods for the variational inequality problem*, Ph.D. Thesis, Instituto de Mathematica Pura e Aplicada (IMPA), Rio de Janeiro, 1995.
[21] R.S. Burachik, S. Scheimberg, *A proximal point method for the variational inequality problem in Banach spaces*, SIAM J. Control Optim., **39**(2000), 1633-1649.
[22] Y. Censor, A. Lent, *An iterative row-action method for interval convex programming*, J. Optim. Theory Appl., **34**(1981), 321-353.
[23] J.W. Chen, Z. Wan, L. Yuan et al., *Approximation of fixed points of weak Bregman relatively nonexpansive mappings in Banach spaces*, Int. J. Math. Math. Sci., (2011), 1-23.
[24] P. Cholamjiak, Y.J. Cho, S. Suantai, *Composite iterative schemes for maximal monotone operators in reflexive Banach spaces*, Fixed Point Theory Appl., 2011.
[25] P. Cholamjiak, S. Suantai, *Convergence analysis for a system of equilibrium problems and a countable family of relatively quasi-nonexpansive mappings in Banach spaces*, Abstr. Appl. Anal., vol. 2010, Article ID 141376, 17 pages.
[26] P.L. Combettes, S.A. Hirstoaga, *Equilibrium programming in Hilbert spaces*, J. Nonlinear Convex Anal., **6**(2005), 117-136.
[27] I. Eckstein, *Nonlinear proximal point algorithms using Bregman function, with applications to convex programming*, Math. Oper. Res., **18**(1993), 202-226.

[28] F. Giannessi, A. Maugeri, P.M. Pardalos (Eds.), *Equilibrium Problems: Nonsmooth Optimization and Variational Inequality Models*, Springer, **58**(2002).
[29] O. Güler, *On the convergence of the proximal point algorithm for convex minimisation*, SIAM J. Control Optim., **29**(1991), 403-419.
[30] S. Kamimura, W. Takahashi, *Approximating solutions of maximal monotone operators in Hilbert spaces*, J. Approx. Theory, **106**(2000), 226-240.
[31] K.C. Kiwiel, *Proximal minimization methods with generalized Bregman functions*, SIAM J. Control Optim., **35**(1997), 1142-1168.
[32] F. Kohsaka, W. Takahashi, *Proximal point algorithms with Bregman functions in Banach spaces*, J. Nonlinear Convex Anal., **6**(2005), 505-523.
[33] Y. Liu, *A general iterative method for equilibrium problems and strict pseudo-contractions in Hilbert spaces*, Nonlinear Anal., **71**(2009), 4852-4861.
[34] P.E. Maingé, *Strong convergence of projected subgradient methods for nonsmooth and non-strictly convex minimization*, Set-Valued Anal., **16**(2008), 899-912.
[35] B. Martinet, *Régularisation d'inéquations variationelles par approximations successives*, Rev. Francaise d'Informatique et de Recherche Opérationelle, **4**(1970), 154-159.
[36] V. Martin-Márquez, S. Reich, S. Sabach, *Right Bregman nonexpansive operators in Banach spaces*, Nonlinear Anal., **75**(2012), 5448-5465.
[37] V. Martin-Márquez, S. Reich, S. Sabach, *Iterative methods for approximating fixed points of Bregman nonexpansive operators*, Discrete and Continuous Dynamical Systems S-series, **6**(2013), no. 4, 1043-1063.
[38] G.B. Passty, *Ergodic convergence to zero of the sum of monotone operators in Hilbert space*, J. Math. Anal. Appl., **72**(1979), 383-390.
[39] R.P. Phelps, *Convex Functions, Monotone Operators, and Differentiability*, 2nd Edition, in: Lecture Notes in Mathematics, vol. 1364, Springer Verlag, Berlin, 1993.
[40] X. Qin, S.M. Kang, Y.J. Cho, *Convergence theorems on generalized equilibrium problems and fixed point problems with applications*, Proc. Estonian Acad. Sci., **58**(2009), 170-318.
[41] X. Qin, Y.J. Cho, S.M. Kang, *Convergence theorems of common elements for equilibrium problems and fixed point problems in Banach spaces*, J. Comput. Appl. Math., **225**(2009), 20-30.
[42] S. Reich, *A weak convergence theorem for the alternating method with Bregman distances*, Theory and Applications of Nonlinear Operators of Accretive and Monotone Type, Lecture Notes in Pure and Appl. Math., vol. 178, Dekker, New York, 1996, 313-318.
[43] S. Reich, S. Sabach, *A strong convergence theorem for a proximal-type algorithm in reflexive Bnanch spaces*, J. Nonlinear Convex Anal., **10**(2009), 471-485.
[44] S. Reich, S. Sabach, *Two strong convergence theorems for a proximal method in reflexive Banach spaces*, Numer. Funct. Anal. Optim., **31**(2010), 22-44.
[45] S. Reich, S. Sabach, *Two strong convergence theorems for Bregman strongly nonexpansive operators in reflexive Banach spaces*, Nonlinear Anal., **73**(2010), 122-135.
[46] S. Reich, S. Sabach, *Existence and approximation of fixed points of Bregman firmly nonexpansive mappings in reflexive Banach spaces*, in: Fixed-Point Algorithms for Inverse Problems in Science and Engineering, Springer, New York, 2011, 299-314.
[47] S. Reich, S. Sabach, *A projection method for solving nonlinear problems in reflexive Banach spaces*, J. Fixed Point Theory Appl., **9**(2011), 101-116.
[48] E. Resmerita, *On total convexity, Bregman projections and stability in Banach spaces*, J. Convex Anal., **11**(2004), 1-16.
[49] R.T. Rockafellar, *Monotone operators and the proximal point algorithm*, SIAM J. Control Optim., **14**(1976), 877-898.
[50] M.V. Solodov, B.F. Svaiter, *Forcing strong convergence of proximal point iterations in a Hilbert space*, Math. Program., **87**(2000), 189-202.
[51] M.V. Solodov, B.F. Svaiter, *An inexact hybrid generalized proximal point algorithm and some new results on the theory of Bregman functions*, Math. Oper. Res., **25**(2000), 214-230.
[52] S. Suantai, Y.J. Cho, P. Cholamjiak, *Halpern's iteration for Bregman strongly nonexpansive mappings in reflexive Banach spaces*, Comp. Math. Appl., **64** (2012), 489-499.

[53] H.K. Xu, *Iterative algorithms for nonlinear operators*, J. London Math. Soc., **66**(2002), no. 2, 240-256.
[54] S. Zhang, *Generalized mixed equilibrium problem in Banach spaces*, Applied Math. Mechanics (English Edition), **30**(2009), 1105-1112.

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