

GENERAL COMPOSITE ITERATIVE METHODS FOR GENERAL SYSTEMS OF VARIATIONAL INEQUALITIES

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Abstract. In this paper, we introduce a general composite implicit scheme and a general composite explicit scheme for finding a solution of general system of variational inequalities in a real Hilbert space. We establish the strong convergence of these two general composite schemes to a solution of the general system of variational inequalities which is the unique solution of some variational inequality. Applications to variational inequalities are given.

Key Words and Phrases: General composite iterative method, general system of variational inequalities, inverse-strongly monotone mapping, strictly pseudocontractive mapping, nonexpansive mapping, fixed point.

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