

## THE MONOTONE MINORANT METHOD AND EIGENVALUE PROBLEM FOR MULTIVALUED OPERATORS IN CONES

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**Abstract.** The main aim of this paper is to obtain a general theorem on existence of continuous branch of solutions of equations which depend on a parameter by using the monotone minorant method in conjunction with the theory of fixed point index. As an application, we apply this theorem to prove the existence of a positive eigen-pair of multivalued homogeneous increasing operators. The simplicity and uniqueness of the eigen-pair are also investigated in this paper.

**Key Words and Phrases:** Cone, positive eigen-pair, fixed point index, monotone minorant, multivalued increasing operator.

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