SOLUTION OF A PAIR OF NONLINEAR MATRIX EQUATIONS

SK MONOWAR HOSSEIN*, SNEHASISH BOSE** AND KALLOL PAUL**

*Department of Mathematics, Aliah University
IIA/27 Newtown, Kolkata - 156, West Bengal, India
E-mail: sami_milu@yahoo.co.uk

**Department of Mathematics, Jadavpur University
Jadavpur-32, West Bengal, India
E-mail: boseasonay@gmail.com, kalloldada@gmail.com

Abstract. In this paper we consider a pair of nonlinear matrix equations of the form $X = Q_1 + (Y^*XY)^{r_1}$, $Y = Q_2 + (X^*YX)^{r_2}$, where $Q_1, Q_2$ are $n \times n$ Hermitian positive definite matrices, $r_1, r_2 \in \mathbb{R}$ and prove the existence and uniqueness of positive definite solutions of these equations. We provide an algorithm to approach the solution. We present a coupled fixed point theorem for non-decreasing mapping and show that a particular case of our nonlinear matrix equations also can be solved by using the derived coupled fixed point theorem. Also we show that by replacing $Y$ with $Y^{-1}$ in first equation and $X$ with $X^{-1}$ in second equation and taking $Q_1 = Q_2$ and $r_1 = r_2$, the reduced system can be solved using the coupled fixed point theorem of Berinde [5].

Key Words and Phrases: Fixed point, partially ordered set, matrix equation, Thompson metric.

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