***Fixed Point Theory***, 19(2018), No. 1, 265-274
DOI 10.24193/fpt-ro.2018.1.21
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

# SOLUTION OF A PAIR OF NONLINEAR MATRIX EQUATIONS

SK MONOWAR HOSSEIN*, SNEHASISH BOSE** AND KALLOL PAUL**

*Department of Mathematics, Aliah University
IIA/27 Newtown, Kolkata - 156, West Bengal, India
E-mail: sami_milu@yahoo.co.uk

**Department of Mathematics, Jadavpur University
Jadavpur-32, West Bengal, India
E-mail: bosesonay@gmail.com, kalloldada@gmail.com

**Abstract.** In this paper we consider a pair of nonlinear matrix equations of the form $X = Q_1 + (Y^*XY)^{r_1}$, $Y = Q_2 + (X^*YX)^{r_2}$, where $Q_1, Q_2$ are $n \times n$ Hermitian positive definite matrices, $r_1, r_2 \in \mathbb{R}$ and prove the existence and uniqueness of positive definite solutions of these equations. We provide an algorithm to approach the solution. We present a coupled fixed point theorem for non-decreasing mapping and show that a particular case of our nonlinear matrix equations also can be solved by using the derived coupled fixed point theorem. Also we show that by replacing $Y$ with $Y^{-1}$ in first equation and $X$ with $X^{-1}$ in second equation and taking $Q_1 = Q_2$ and $r_1 = r_2$, the reduced system can be solved using the coupled fixed point theorem of Berinde [5].
**Key Words and Phrases**: Fixed point, partially ordered set, matrix equation, Thompson metric.
**2010 Mathematics Subject Classification**: 15A24, 47H10, 47H09.

**Acknowledgments.** Sk. Monowar Hossein would like to thank IMSC, Chennai, India for providing research facilities. Snehasish Bose gratefully acknowledges the financial support of CSIR, Govt. of India. The authors also gratefully acknowledge the reviewer for his/her valuable suggestions.

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*Received: September 2, 2015; Accepted: February 21, 2016.*