Fixed Point Theory, 19(2018), No. 1, 249-264
DOI 10.24193/fpt-ro.2018.1.20
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

POSITIVE SOLUTIONS FOR A SYSTEM OF SINGULAR SECOND-ORDER INTEGRAL BOUNDARY VALUE PROBLEMS

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Abstract. We investigate the existence of positive solutions of a system of second-order nonlinear differential equations subject to integral boundary conditions, where the nonlinearities do not possess any sublinear or superlinear growth conditions and may be singular.
Key Words and Phrases: System of second-order differential equations, integral boundary conditions, positive solutions, singular functions, fixed point index.
2010 Mathematics Subject Classification: 34B10, 34B18, 47H10.

Acknowledgement. The authors thank the referee for his/her valuable comments and suggestions. The work of R. Luca was supported by a grant of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, project number PN-II-ID-PCE-2011-3-0557.

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Received: October 8, 2015; Accepted: June 28, 2016.