GENERAL ALGORITHM FOR EQUILIBRIUM PROBLEMS AND SET-VALUED OPERATORS

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Abstract. In this paper we introduce and study a general algorithm to approximate a common element of the set of solutions of a system of equilibrium problems and the set of common fixed points of an infinite family of quasi-nonexpansive set-valued mappings. We prove strong convergence of such algorithm in a real Hilbert space. This common solution is the unique solution of a variational inequality problem and is the optimality condition for a minimization problem. Our results improve and extend many related results in the literature.

Key Words and Phrases: Equilibrium problem, set-valued mapping, variational inequality, quasi-nonexpansive mapping, common fixed point.

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